



Introductory Algebra

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To Dotty, Puddles, and Gus. You loved us unconditionally. E.J.H. and T.R.M.

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Preface

It is with great pleasure that we offer the eleventh edition of *Introductory Algebra*. We have remained true to the original goal that has guided us over the years—to provide the best possible text and supplements package to help students succeed and instructors teach. This edition faithfully continues that process through enhanced explanations of concepts, new and updated examples and exercises, student-oriented features like Pointers, Cautions, Problem-Solving Hints, Margin Problems, and Study Skills, as well as an extensive package of helpful supplements and study aids.

This text is part of a series that also includes the following books:

- Basic College Mathematics, Tenth Edition, by Lial, Salzman, and Hestwood
- Prealgebra, Sixth Edition, by Lial and Hestwood
- Intermediate Algebra, Eleventh Edition, by Lial, Hornsby, and McGinnis
- Introductory and Intermediate Algebra, Sixth Edition, by Lial, Hornsby, and McGinnis
- Developmental Mathematics: Basic Mathematics and Algebra, Fourth Edition, by Lial, Hornsby, McGinnis, Salzman, and Hestwood

WHAT'S NEW IN THIS EDITION

We are pleased to offer the following new textbook features and supplements.

- ▶ *Revised Exposition* With each edition of the text, we continue to polish and improve discussions and presentations of topics to increase readability and student understanding. We believe this edition is the best yet in this regard.
- ▶ *More Figures and Diagrams* For visual learners, we have made a concerted effort to add mathematical figures, diagrams, tables, and graphs whenever possible.
- ▶ *Improved Study Skills* Most of these special activities now include a *Now Try This* section to increase student involvement. Each is designed independently to allow flexible use with individuals or small groups of students, or as a source of material for in-class discussions.
- ► *More What Went Wrong? Exercises* We have increased the number of these popular CONCEPT CHECK exercises, which highlight common student errors.
- ▶ More Relating Concepts Exercises We have increased the number of these flexible groups of exercises, located at the end of many exercise sets. Specially written to help students tie concepts together, as well as compare and contrast ideas, identify and describe patterns, and extend concepts to new situations, these sets of problems may be used with individual students or collaboratively with pairs or small groups.
- ▶ Dedicated Mixed Review Exercises Each chapter review has been expanded to include a one-page set of Mixed Review Exercises to help students further synthesize concepts.
- ▶ Learning Catalytics This interactive student response tool uses students' own devices to engage them in the learning process. Learning Catalytics is accessible through MyMathLab and can be customized to an instructor's specific needs. Instructors can employ this tool to generate class discussion, promote peer-to-peer learning, and use real-time data to adjust instructional strategy. As an introduction to this exciting new tool, we have provided prerequisite skills questions at the beginning of each section to check students' preparedness for the new section. Learn more about Learning Catalytics in the Instructor Resources tab in MyMathLab.

▶ Data Analytics We analyzed aggregated student usage and performance data from MyMathLab for the previous edition of this text. The results of this analysis helped us improve the quality and quantity of exercises that matter the most to instructors and students.

CONTENT CHANGES

Specific content changes include the following:

- ► Exercise sets have been updated with a renewed focus on conceptual understanding, skill development, and review. New or revised figures are included wherever possible.
- ▶ **Real-world data** in the examples and exercises has been updated.
- ▶ More "word equations" are included in application examples to help students translate words into equations.
- ▶ Expanded Chapter R includes new figures and exposition on fractions, as well as new discussion, examples, and exercises on converting between fractions, decimals, and percents.
- ▶ Expanded Mid-Chapter Summary Exercises in Chapter 2 continue our emphasis on the difference between simplifying expressions and solving equations. The mid-chapter Summary Exercises in Chapters 4, 6, and 9 include new examples that illustrate and distinguish between solution methods.
- ▶ Separate sections on slope-intercept form and point-slope form now appear in Chapter 3 and include enhanced discussion and new examples and exercises.
- ▶ Reorganized Chapter 5 introduces the rules for exponents and application to scientific notation at the beginning of the chapter, followed by the sections on polynomials and their operations.
- ▶ The following topics are among those that have been enhanced and/or expanded:

Operations with signed numbers (Sections 1.4–1.6)

Order of operations involving absolute value expressions (Sections 1.5 and 1.6)

Solving linear equations in one variable (Sections 2.1 and 2.2)

Solving linear inequalities with fractions (Section 2.7)

Graphing linear equations in two variables using intercepts (Section 3.2)

Solving linear systems of equations using elimination (Section 4.3)

Dividing a polynomial by a polynomial (Section 5.7)

Discussion of sums of squares and factoring perfect square trinomials (Section 6.5)

Solving variation problems (Section 7.8)

Adding and subtracting radicals (Section 8.3)

Solving quadratic equations by completing the square (Section 9.2)

HALLMARK FEATURES

We have enhanced the following popular features, each of which is designed to increase ease-of-use by students and/or instructors.

- ► Emphasis on Problem-Solving We introduce our six-step problem-solving method in Chapter 2 and integrate it throughout the text. The six steps, Read, Assign a Variable, Write an Equation, Solve, State the Answer, and Check, are emphasized in boldface type and repeated in examples and exercises to reinforce the problem-solving process for students. We also provide students with problem-solving tips and strategies.

 Problem-Solving Hint boxes that feature helpful problem-solving tips and strategies.
- ▶ *Helpful Learning Objectives* We begin each section with clearly stated, numbered objectives, and the included material is directly keyed to these objectives so that students and instructors know exactly what is covered in each section.

- ▶ *Popular Cautions and Notes* One of the most popular features of previous editions, we include information marked ① CAUTION and Note to warn students about common errors and emphasize important ideas throughout the exposition. The updated text design makes them easy to spot.
- ► Comprehensive Examples The new edition features a multitude of step-by-step, worked-out examples that include pedagogical color, helpful side comments, and special pointers. We give special attention to checking example solutions—more checks, designated using a special CHECK tag and ✓, are included than in past editions.
- ▶ *More Pointers* Because they were so well received by both students and instructors in the previous edition, we incorporate more pointers in examples and discussions throughout this edition of the text. They provide students with important on-the-spot reminders and warnings about common pitfalls.
- ▶ Ample Margin Problems Margin problems, with answers immediately available at the bottom of the page, are found in every section of the text. This key feature allows students to immediately practice the material covered in the examples in preparation for the exercise sets. Many include guided solutions.
- ▶ Updated Figures, Photos, and Hand-Drawn Graphs Today's students are more visually oriented than ever. As a result, we include appealing mathematical figures, diagrams, tables, and graphs, including a "hand-drawn" style of graphs, whenever possible. We have incorporated depictions of well-known mathematicians as well as photos to accompany applications in examples and exercises.
- ▶ *Relevant Real-Life Applications* We include many new or updated applications from fields such as business, pop culture, sports, technology, and the health sciences that show the relevance of algebra to daily life.
- ▶ Extensive and Varied Exercise Sets The text contains a wealth of exercises to provide students with opportunities to practice, apply, connect, review, and extend the skills they are learning. Numerous illustrations, tables, graphs, and photos help students visualize the problems they are solving. Problem types include skill building and writing exercises, as well as applications, matching, true/false, multiple-choice, and fill-in-the-blank problems.
 - In the Annotated Instructor's Edition of the text, the writing exercises are marked with an icon \nearrow so that instructors may assign these problems at their discretion. Students can watch an instructor work through the complete solution for all exercises marked with a Play Button icon \bigcirc in MyMathLab.
- ▶ Special Summary Exercises We include a set of these popular in-chapter exercises in every chapter beginning in Chapter 1. They provide students with the all-important mixed review problems they need to master topics and often include summaries of solution methods and/or additional examples.
- ▶ Step-by-Step Solutions to Selected Exercises Exercise numbers enclosed in a blue square, such as 11., indicate that a worked-out solution for the problem is available in MyMathLab. These solutions are given for selected exercises that most commonly cause students difficulty.

Resources for Success

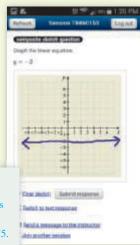
Pearson

MyMathLab Online Course for Lial/Hornsby/ McGinnis *Introductory Algebra*, 11th edition

The corresponding MyMathLab course tightly integrates the authors' approach, giving students a learning environment that encourages conceptual understanding and engagement.

NEW! Learning Catalytics

Integrated into MyMathLab, Learning Catalytics use students' mobile devices for an engagement, assessment, and classroom intelligence system that gives instructors real-time feedback on student learning. LC annotations for instructors in the text provide corresponding questions that they can use to engage their classrooms.



LEARNING CATALYTICS

- **1.** Which digit in the number 40,163 is in the ones place?
- **2.** Identify the place value of 8 in the number 9875.



NEW! Workspace Assignments

These new assignments allow students to naturally write out their work by hand, step-by-step, showing their mathematical reasoning as they receive instant feedback at each step. Each student's work is captured in the MyMathLab gradebook so instructors can easily pinpoint exactly where in the solution process students struggled.

Expanded! Conceptual Exercises

In addition to MyMathLab's hallmark interactive exercises, the Lial team provides students with exercises that tie concepts together and help students problem-solve. Guided Solutions exercises, marked with a "GS" in the Assignment Manager, test student understanding of the problem-solving steps while guiding them through the solution process. Relating Concepts exercises in the text help students make connections and problem-solve at a higher level. These sets are assignable in MyMathLab, with expanded coverage.

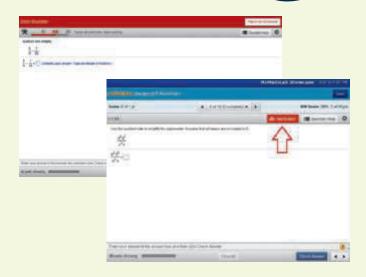


Resources for Success



NEW! Adaptive Skill Builder

When students struggle on an exercise, Skill Builder assignments provide just-in-time, targeted support to help them build on the requisite skills needed to complete their assignment. As students progress, the Skill Builder assignments adapt to provide support exercises that are personalized to each student's activity and performance throughout the course.



Instructor Resources

Annotated Instructor's Edition

ISBN 10: 0-13-449580-2 **ISBN 13:** 978-0-13-449580-4 The AIE provides annotations for instructors, including answers, Learning Catalytics suggestions, and vocabulary and teaching tips.

The following resources can be downloaded from www.pearsonhighered.com or in MyMathLab:

Instructor's Solutions Manual

This manual provides solutions to all exercises in the text.

Instructor's Resource Manual

This manual includes Mini-Lectures to provide new instructors with objectives, key examples, and teaching tips for every section of the text.

PowerPoints

These slides, which can be edited, present key concepts and definitions from the text.

TestGen

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

Student Resources

Student Solutions Manual

ISBN 10: 0-13- 450922-6 **ISBN 13:** 978-0-13- 450922-8 This manual contains completely worked-out solutions for all the odd-numbered exercises in the text.

Lial Video Workbook

ISBN 10: 0-13-450923-4 **ISBN 13:** 978-0-13-450923-5 This workbook/note-taking guide helps students develop organized notes as they work along with the videos. The notebook includes

- Guided Examples to be used in conjunction with the Lial Section Lecture Videos and/or Objective-Level Video clips, plus corresponding Now Try This Exercises for each text objective.
- Extra practice exercises for every section of the text, with ample space for students to show their work.
- Learning objectives and key vocabulary terms for every text section, along with vocabulary practice problems.

ACKNOWLEDGMENTS

The comments, criticisms, and suggestions of users, nonusers, instructors, and students have positively shaped this text over the years, and we are most grateful for the many responses we have received. The feedback gathered for this revision of the text was particularly helpful, and we especially wish to thank the following individuals who provided invaluable suggestions for this and the previous editions:

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We particularly thank the many students and instructors who have used this text over the years. You are the reason we do what we do. It is our hope that we have positively impacted your mathematics journey. We would welcome any comments or suggestions you might have via email to math@pearson.com.

John Hornsby Terry McGinnis



Prealgebra Review

R.1 Fractions

R.2 Decimals and Percents

Study Skills Using Your Math Text

Fractions

The numbers used most often in everyday life are the **natural** (**counting**) numbers,

$$1, 2, 3, 4, \ldots,$$

the whole numbers,

The three dots, or *ellipsis points*, indicate that each list of numbers continues in the same way indefinitely.

and fractions, such as

$$\frac{1}{2}$$
, $\frac{2}{3}$, and $\frac{11}{12}$.

The parts of a fraction are named as follows.

Fraction bar
$$\rightarrow \frac{3}{8}$$
 \leftarrow Numerator \leftarrow Denominator

The fraction bar represents division $(\frac{a}{b} = a \div b)$.

Note

Fractions are a way to represent parts of a whole. In a fraction, the **numerator** gives the number of parts being represented. The **denominator** gives the total number of equal parts in the whole. See Figure 1.



The shaded region represents $\frac{3}{8}$ of the circle.

Figure 1

A fraction is classified as being either a **proper fraction** or an **improper** fraction.

Proper fractions
$$\frac{1}{5}$$
, $\frac{2}{7}$, $\frac{9}{10}$, $\frac{23}{25}$

Numerator is **less than** denominator. Value is less than 1.

Improper fractions $\frac{3}{2}$, $\frac{5}{5}$, $\frac{11}{7}$, $\frac{28}{4}$

Numerator is greater than or equal to denominator. Value is

greater than or equal to 1.

OBJECTIVES

- Identify prime numbers.
- Write numbers in prime factored
- Write fractions in lowest terms.
- 4 Convert between improper fractions and mixed numbers.
- Multiply and divide fractions.
- 6 Add and subtract fractions.
- Solve applied problems that involve fractions.
- 8 Interpret data in a circle graph.

- 1 Identify each number as *prime* or *composite*.
 - **(a)** 13
 - **(b)** 27
 - **(c)** 59
 - (d) 1806
- 2 Write each number in prime factored form.
 - (a) 39
 - **(b)** 70
 - (c) 72
 - **(d)** 135

Answers

- 1. (a) prime (b) composite (c) prime (d) composite
- **2.** (a) $3 \cdot 13$ (b) $2 \cdot 5 \cdot 7$ (c) $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$ (d) $3 \cdot 3 \cdot 3 \cdot 5$

OBJECTIVE 1 Identify prime numbers. In work with fractions, we will need to write numerators and denominators as *products*. A **product** is the answer to a multiplication problem. When 12 is written as the product 2×6 , for example, 2 and 6 are **factors** of 12. Other factors of 12 are 1, 3, 4, and 12.

A natural number greater than 1 is **prime** if it has only itself and 1 as factors. "Factors" are understood here to mean natural number factors.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 First dozen prime numbers

A natural number greater than 1 that is not prime is a **composite number.**

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21 First dozen composite numbers

By agreement, the number 1 is neither prime nor composite.

EXAMPLE 1 Distinguishing between Prime and Composite Numbers

Identify each number as prime or composite.

- (a) 33 Since 33 has factors of 3 and 11, as well as 1 and 33, it is composite.
- (b) 43 There are no numbers other than 1 and 43 itself that divide *evenly* into 43, so the number 43 is prime.
- (c) 9832 Since 9832 can be divided by 2, giving 2×4916 , it is composite.

── ■ Work Problem 1 at the Side.

OBJECTIVE 2 Write numbers in prime factored form. We *factor* a number by writing it as the product of two or more numbers.

MultiplicationFactoring
$$6 \cdot 3 = 18$$
 $18 = 6 \cdot 3$ $\uparrow \uparrow \uparrow \uparrow$ $\uparrow \uparrow \uparrow$ Factors ProductProduct Factors

Factoring is the reverse of multiplying two numbers to obtain the product.

Note

In algebra, a raised dot \cdot is often used instead of the \times symbol to indicate multiplication because \times may be confused with the letter x.

A composite number written using factors that are all prime numbers is in **prime factored form.**

EXAMPLE 2 Writing Numbers in Prime Factored Form

Write each number in prime factored form.

- (a) 35 We factor 35 using the prime factors 5 and 7 as $35 = 5 \cdot 7$.
- **(b)** 24 We use a factor tree, as shown below. The prime factors are circled.

Divide by the least prime factor of 24, which is 2. $24 = 2 \cdot 12$ Divide 12 by 2 to find two factors of 12. $24 = 2 \cdot 2 \cdot 6$

Now factor 6 as $2 \cdot 3$.

 $24 = 2 \cdot 2 \cdot 2 \cdot 3$

All factors are prime.

— ■ Work Problem ② at the Side.

Note

No matter which prime factor we start with when factoring, we will *always* obtain the same prime factorization. Verify that if we start with 3 instead of 2 in **Example 2(b)**, we obtain

$$24 = 3 \cdot 2 \cdot 2 \cdot 2$$
.

The order of the factors is different, but the same prime factors result.

OBJECTIVE 3 Write fractions in lowest terms. The following properties are useful when writing a fraction in *lowest terms*.

Properties of 1

Any nonzero number divided by itself is equal to 1. Example: $\frac{3}{3} = 1$

Any number multiplied by 1 remains the same. Example: $\frac{2}{5} \cdot 1 = \frac{2}{5}$

A fraction is in **lowest terms** when the numerator and denominator have no factors in common (other than 1).

Writing a Fraction in Lowest Terms

- **Step 1** Write the numerator and denominator in factored form.
- **Step 2** Replace each pair of factors common to the numerator and denominator with 1.
- **Step 3** Multiply the remaining factors in the numerator and in the denominator.

(This procedure is sometimes called "simplifying the fraction.")

EXAMPLE 3 Writing Fractions in Lowest Terms

Write each fraction in lowest terms.

(a)
$$\frac{10}{15} = \frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3} \cdot \frac{5}{5} = \frac{2}{3} \cdot 1 = \frac{2}{3}$$
 Use the first property of 1 to replace $\frac{5}{5}$ with 1.

(b)
$$\frac{15}{45}$$
 By inspection, the greatest common factor of 15 and 45 is 15.

$$\frac{15}{45} = \frac{15}{3 \cdot 15} = \frac{1}{3 \cdot 1} = \frac{1}{3}$$
 Remember to write 1 in the numerator.

If the greatest common factor is not obvious, factor the numerator and denominator into prime factors.

$$\frac{15}{45} = \frac{3 \cdot 5}{3 \cdot 3 \cdot 5} = \frac{1 \cdot 1}{3 \cdot 1 \cdot 1} = \frac{1}{3}$$
 The same answer results.

(c)
$$\frac{150}{200} = \frac{3 \cdot 50}{4 \cdot 50} = \frac{3}{4} \cdot 1 = \frac{3}{4}$$
 50 is the greatest common factor of 150 and 200.

Another strategy is to choose any common factor and work in stages.

$$\frac{150}{200} = \frac{15 \cdot 10}{20 \cdot 10} = \frac{3 \cdot 5 \cdot 10}{4 \cdot 5 \cdot 10} = \frac{3 \cdot 1 \cdot 1}{4 \cdot 1 \cdot 1} = \frac{3}{4}$$
 The same answer results.

Work Problem 3 at the Side. ▶

- 3 Write each fraction in lowest terms.
 - (a) $\frac{8}{14}$

(b) $\frac{10}{70}$

(c) $\frac{72}{120}$

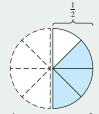
Answers

3. (a) $\frac{4}{7}$ (b) $\frac{1}{7}$ (c) $\frac{3}{5}$

4 Write $\frac{92}{5}$ as a mixed number.

5 Write $11\frac{2}{3}$ as an improper fraction.

Multiplying Fractions



 $\frac{3}{4}$ of $\frac{1}{2}$ is equivalent to $\frac{3}{4} \cdot \frac{1}{2}$, which equals $\frac{3}{8}$ of the circle.

Figure 2

Answers

4.
$$18\frac{2}{5}$$

5.
$$\frac{35}{3}$$

OBJECTIVE 4 Convert between improper fractions and mixed numbers. A mixed number is a single number that represents the sum of a natural number and a proper fraction.

Mixed number
$$\rightarrow 5\frac{3}{4} = 5 + \frac{3}{4}$$

EXAMPLE 4 Converting an Improper Fraction to a Mixed Number

Write $\frac{59}{8}$ as a mixed number.

The fraction bar represents division. We divide the numerator of the improper fraction by the denominator.

Denominator of fraction
$$8\overline{\smash)59}$$
 \leftarrow Quotient 59 $8\overline{\smash)59}$ \leftarrow Numerator of fraction (dividend) $8\overline{\smash)59}$ \leftarrow Remainder $8\overline{\smash)59}$ \leftarrow Remainder

── ■ Work Problem 4 at the Side.

EXAMPLE 5 Converting a Mixed Number to an Improper Fraction

Write $6\frac{4}{7}$ as an improper fraction.

We multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.

$$7 \cdot 6 = 42$$
 and $42 + 4 = 46$

The denominator of the improper fraction is the same as the denominator in the mixed number, which is 7 here. Thus, $6\frac{4}{7} = \frac{46}{7}$.

OBJECTIVE 5 Multiply and divide fractions. See Figure 2.

Multiplying Fractions

To multiply two fractions, multiply the numerators to obtain the numerator of the product. Multiply the denominators to obtain the denominator of the product. *The product should be written in lowest terms*.

EXAMPLE 6 Multiplying Fractions

Find each product, and write it in lowest terms as needed.

(a)
$$\frac{3}{8} \cdot \frac{4}{9}$$

$$= \frac{3 \cdot 4}{8 \cdot 9}$$
Multiply numerators.
Multiply denominators.
$$= \frac{3 \cdot 4}{2 \cdot 4 \cdot 3 \cdot 3}$$
Factor the denominator.
$$= \frac{1}{2 \cdot 3}$$
Remember to write 1 in the numerator.
$$= \frac{1}{6}$$
Write in lowest terms.

———— Continued on Next Page

(b)
$$2\frac{1}{3} \cdot 5\frac{1}{4}$$

$$= \frac{7}{3} \cdot \frac{21}{4}$$
Write each mixed number as an improper fraction.
$$= \frac{7 \cdot 21}{3 \cdot 4}$$
Multiply numerators.
Multiply denominators.
$$= \frac{7 \cdot 3 \cdot 7}{3 \cdot 4}$$
Factor the numerator.

Think: $\frac{49}{4}$ means $49 \div 4$.
$$= \frac{49}{4}$$
, or $12\frac{1}{4}$ Write in lowest terms and as a mixed number.

Work Problem 6 at the Side.

Two fractions are **reciprocals** of each other if their product is 1. See the table.

RECIPROCALS

Number	Reciprocal
3/4	$\frac{4}{3}$
<u>11</u> 7	7 11
<u>1</u> 5	5, or $\frac{5}{1}$
9, or $\frac{9}{1}$	<u>1</u>

Example: $\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1$

Because division is the inverse, or opposite, of multiplication, we use reciprocals to divide fractions.

Figure 3 illustrates dividing fractions.

Dividing Fractions

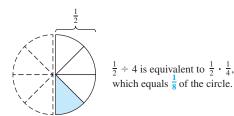


Figure 3

Dividing Fractions

To divide two fractions, multiply the first fraction by the reciprocal of the second. The result or **quotient** should be written in lowest terms.

As an example of why this procedure works, we know that

$$20 \div 10 = 2$$
 and also that $20 \cdot \frac{1}{10} = 2$.

6 Find each product, and write it in lowest terms as needed.

(a)
$$\frac{5}{6} \cdot \frac{3}{10}$$

(b)
$$\frac{4}{7} \cdot \frac{5}{8}$$

(c)
$$3\frac{1}{3} \cdot 1\frac{3}{4}$$

Answers

6. (a)
$$\frac{1}{4}$$
 (b) $\frac{5}{14}$ (c) $\frac{35}{6}$, or $5\frac{5}{6}$

7 Find each quotient, and write it in lowest terms as needed.

(a)
$$\frac{2}{7} \div \frac{3}{10}$$

(b)
$$\frac{3}{4} \div \frac{7}{16}$$

(c)
$$\frac{4}{3} \div 6$$

(d)
$$3\frac{1}{4} \div 1\frac{2}{5}$$

Answers

7. (a)
$$\frac{20}{21}$$
 (b) $\frac{12}{7}$, or $1\frac{5}{7}$ (c) $\frac{2}{9}$ (d) $\frac{65}{28}$, or $2\frac{9}{28}$

EXAMPLE 7 Dividing Fractions

Find each quotient, and write it in lowest terms as needed.

(a)
$$\frac{3}{4} \div \frac{8}{5}$$

$$= \frac{3}{4} \cdot \frac{5}{8}$$
Multiply by the reciprocal of the second fraction.
$$= \frac{3 \cdot 5}{4 \cdot 8}$$
Multiply numerators.
$$= \frac{15}{32}$$
Make sure the quotient is in lowest terms.

(b)
$$\frac{3}{4} \div \frac{5}{8}$$
 (c) $\frac{5}{8} \div 10$ Think of 10 as $\frac{10}{1}$ here.

$$= \frac{3}{4} \cdot \frac{8}{5}$$
 Multiply by the reciprocal.

$$= \frac{3 \cdot 4 \cdot 2}{4 \cdot 5}$$
 Multiply and factor.

$$= \frac{6}{5}$$
, or $1\frac{1}{5}$ $= \frac{1}{16}$ Remember to write 1 in the numerator.

(d)
$$1\frac{2}{3} \div 4\frac{1}{2}$$

$$= \frac{5}{3} \div \frac{9}{2}$$
 Write each mixed number as an improper fraction.
$$= \frac{5}{3} \cdot \frac{2}{9}$$
 Multiply by the reciprocal of the second fraction.
$$= \frac{10}{27}$$
 Multiply numerators and denominators.
The quotient is in lowest terms.

■ Work Problem 7 at the Side.

OBJECTIVE 6 Add and subtract fractions. The result of adding two numbers is the sum of the numbers. Figure 4 illustrates adding fractions.

Adding Fractions $\frac{1}{8} + \frac{3}{8}$ $= \frac{4}{8}$ $= \frac{1}{2}$ Figure 4

Adding Fractions

To find the sum of two fractions with the *same* denominator, add their numerators and *keep the same denominator*.

EXAMPLE 8 Adding Fractions (Same Denominator)

Find each sum, and write it in lowest terms as needed.

(a)
$$\frac{3}{7} + \frac{2}{7}$$

(b)
$$\frac{2}{10} + \frac{3}{10}$$

$$=\frac{3+2}{7}$$

Add numerators.

Add numerators.

Keep the same denominator.

Keep the same denominator.

The answer is in lowest terms.

Write in lowest terms.

Work Problem 8 at the Side.

If the fractions to be added do not have the same denominator, we must first rewrite them with a common denominator. For example, to rewrite $\frac{3}{4}$ as a fraction with a denominator of 12, think as follows.

$$\frac{3}{4} = \frac{?}{12}$$

We must find the number that can be multiplied by 4 to give 12. Because $4 \cdot 3 = 12$, by the second property of 1, we multiply the numerator and the denominator by 3.

$$\frac{3}{4} = \frac{3}{4} \cdot 1 = \frac{3}{4} \cdot \frac{3}{3} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$$

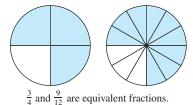


Figure 5

Note

The process of writing an equivalent fraction is the reverse of writing a fraction in lowest terms.

Finding the Least Common Denominator (LCD)

To add or subtract fractions with different denominators, find the least common denominator (LCD) as follows.

- **Step 1** Write each denominator in prime factored form.
- The LCD is the product of every (different) factor that appears in Step 2 any of the factored denominators. If a factor is repeated, use the greatest number of repeats as factors of the LCD.
- Write each fraction with the LCD as the denominator, using the second property of 1.

Find each sum, and write it in lowest terms as needed.

(a)
$$\frac{5}{11} + \frac{3}{11}$$

(b)
$$\frac{5}{14} + \frac{3}{14}$$

(c)
$$\frac{3}{5} + \frac{4}{5}$$

8. (a)
$$\frac{8}{11}$$
 (b) $\frac{4}{7}$ (c) $\frac{7}{5}$, or $1\frac{2}{5}$

9 Find each sum, and write it in lowest terms as needed.

(a)
$$\frac{5}{12} + \frac{3}{8}$$

(b)
$$\frac{7}{30} + \frac{2}{45}$$

(c)
$$2\frac{1}{8} + 1\frac{2}{3}$$

(d)
$$4\frac{5}{6} + 2\frac{1}{3}$$

Answers

9. (a)
$$\frac{19}{24}$$
 (b) $\frac{5}{18}$ (c) $\frac{91}{24}$, or $3\frac{19}{24}$ (d) $\frac{43}{6}$, or $7\frac{1}{6}$

EXAMPLE 9 Adding Fractions (Different Denominators)

Find each sum, and write it in lowest terms as needed.

(a)
$$\frac{4}{15} + \frac{5}{9}$$

Step 1 To find the LCD, write each denominator in prime factored form

$$15 = 5 \cdot 3$$
 and $9 = 3 \cdot 3$
3 is a factor of both denominators.

In this example, the LCD needs one factor of 5 and two factors of 3 because the second denominator has two factors of 3.

Step 3 Now we can use the second property of 1 to write each fraction with 45 as the denominator.

$$\frac{4}{15} = \frac{4}{15} \cdot \frac{3}{3} = \frac{12}{45} \quad \text{and} \quad \frac{5}{9} = \frac{5}{9} \cdot \frac{5}{5} = \frac{25}{45} \quad \text{At this stage, the fractions are not in lowest terms.}$$

$$\frac{4}{15} + \frac{5}{9}$$

$$= \frac{12}{45} + \frac{25}{45}$$
 Use the equivalent fractions with the common denominator.

Make sure the sum is in lowest terms.
$$=\frac{37}{45}$$
 Add numerators. Keep the same denominator.

(b)
$$3\frac{1}{2} + 2\frac{3}{4}$$

Method I
$$3\frac{1}{2} + 2\frac{3}{4}$$

$$= \frac{7}{2} + \frac{11}{4}$$
Write each mixed number as an improper fraction.

Think:
$$\frac{7 \cdot 2}{2 \cdot 2} = \frac{14}{4}$$
 = $\frac{14}{4} + \frac{11}{4}$ Find a common denominator. The LCD is 4.

$$= \frac{25}{4}, \text{ or } 6\frac{1}{4} \text{ Add. Write as a mixed number.}$$

Method 2
$$3\frac{1}{2} = 3\frac{2}{4}$$
 Write $3\frac{1}{2}$ as $3\frac{2}{4}$. Then add vertically. Add the natural numbers and the fractions separately.
$$5\frac{5}{4} = 5 + 1\frac{1}{4} = 6\frac{1}{4}, \text{ or } \frac{25}{4} \text{ The same answer results.}$$

Work Problem 9 at the Side.

The result of subtracting one number from another number is the **difference** of the numbers. Figure 6 illustrates subtracting fractions.

Subtracting Fractions

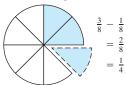


Figure 6

Subtracting Fractions

To find the difference of two fractions with the same denominator, subtract their numerators and keep the same denominator.

If the fractions have different denominators, write them with a common denominator first.

EXAMPLE 10 Subtracting Fractions

Find each difference, and write it in lowest terms as needed.

(a)
$$\frac{15}{8} - \frac{3}{8}$$

$$= \frac{15-3}{8}$$
 Subtract numerators. Keep the same denomination

Keep the same denominator.

$$=\frac{12}{8}$$

$$=\frac{3}{2}$$
, or $1\frac{1}{2}$

 $=\frac{3}{2}$, or $1\frac{1}{2}$ Write in lowest terms and as a mixed number.

(b)
$$\frac{15}{16} - \frac{4}{9}$$

$$=\frac{15}{16} \cdot \frac{9}{9} - \frac{4}{9} \cdot \frac{16}{16}$$

 $= \frac{15}{16} \cdot \frac{9}{9} - \frac{4}{9} \cdot \frac{16}{16}$ Because 16 and 9 have no common factors except 1, the LCD is $16 \cdot 9 = 144$.

$$=\frac{135}{144}-\frac{64}{144}$$

Write equivalent fractions.

$$=\frac{71}{144}$$

Subtract numerators.

Keep the common denominator.

(c)
$$\frac{7}{18} - \frac{4}{15}$$

$$= \frac{7}{2 \cdot 3 \cdot 3} \cdot \frac{5}{5} - \frac{4}{3 \cdot 5} \cdot \frac{2 \cdot 3}{2 \cdot 3}$$
 18 = 2 \cdot 3 \cdot 3 and 15 = 3 \cdot 5, so the LCD is 2 \cdot 3 \cdot 3 \cdot 5 = 90.

$$=\frac{35}{90}-\frac{24}{90}$$

Write equivalent fractions.

$$=\frac{11}{90}$$

Subtract. The answer is in lowest terms.

10 Find each difference, and write it in lowest terms as needed.

(a)
$$\frac{9}{11} - \frac{3}{11}$$

(b)
$$\frac{5}{11} - \frac{2}{9}$$

(c)
$$\frac{13}{15} - \frac{5}{6}$$

(d)
$$2\frac{3}{8} - 1\frac{1}{2}$$

(e)
$$10\frac{1}{4} - 2\frac{2}{3}$$

11 Solve the problem.

To make a three-piece outfit from the same fabric, Jen needs $1\frac{1}{4}$ yd for the blouse, $1\frac{2}{3}$ yd for the skirt, and $2\frac{1}{2}$ yd for the jacket. How much fabric does she need?

10. (a)
$$\frac{6}{11}$$
 (b) $\frac{23}{99}$ (c) $\frac{1}{30}$ (d) $\frac{7}{8}$ (e) $7\frac{7}{12}$

11. $5\frac{5}{12}$ yd

(d)
$$4\frac{1}{2} - 1\frac{3}{4}$$

$$Method 1 \qquad 4\frac{1}{2} - 1\frac{3}{4}$$

$$=\frac{9}{2}-\frac{7}{4}$$

Write each mixed number as an improper fraction.

Think:
$$\frac{9 \cdot 2}{2 \cdot 2} = \frac{18}{4}$$

$$= \frac{18}{4} - \frac{7}{4}$$

Find a common denominator. The LCD is 4.

$$=\frac{11}{4}$$
, or $2\frac{3}{4}$ Subtract. Write as a mixed number.

Method 2
$$4\frac{1}{2} = 4$$

$$4\frac{1}{2} = 4\frac{2}{4} = 3\frac{6}{4}$$

Method 2 $4\frac{1}{2} = 4\frac{2}{4} = 3\frac{6}{4}$ The LCD is 4. $4\frac{2}{4} = 3 + 1 + \frac{2}{4} = 3 + \frac{4}{4} + \frac{2}{4} = 3\frac{6}{4}$

$$-1\frac{3}{4} = 1\frac{3}{4} = 1\frac{3}{4}$$

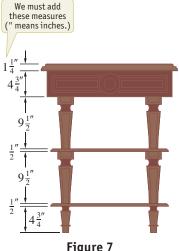
 $2\frac{3}{4}$, or $\frac{11}{4}$ The same answer results.

◄ Work Problem **10** at the Side.

OBJECTIVE 7 Solve applied problems that involve fractions.

EXAMPLE 11 Solving an Applied Problem with Fractions

The diagram in **Figure 7** appears with directions for a woodworking project. Find the height of the desk to the top of the writing surface.



$$1\frac{1}{4} \to 1\frac{1}{4}$$

$$4\frac{3}{4} \to 4\frac{3}{4}$$

 $9\frac{1}{2} = 9\frac{2}{4}$

Add the section and writing $1\frac{1}{4} \rightarrow 1\frac{1}{4}$ surface heights to obtain the total height. The common denominator is 4.

 $\frac{+4\frac{3}{4} \rightarrow 4\frac{3}{4}}{27\frac{15}{4}}$

Because $\frac{15}{4} = 3\frac{3}{4}$, we have $27\frac{15}{4} = 27 + 3\frac{3}{4} = 30\frac{3}{4}$. The height is $30\frac{3}{4}$ in.

───── ■ Work Problem 111 at the Side.